IMPORTANT! **NO CLASS THIS WEDNESDAY**!

- Our February 3 class is rescheduled for March 18 at 9AM (location TBD).
- This week’s ‘in-between’ Lab
- Optional lectures on Brahms

Optional Lectures on Brahms

- As of Tuesday February 2, every Tuesday and Thursday from 1-3PM the Brahms developer, Dr. Maarten Sierhuis – Professor at Carnegie Mellon University, will remotely lecture for us and for students at Delft University of Technology.
- The lectures will be transmitted out of my Adaptive Risk Management Lab located in room H 212. **All students enrolled in this class are welcome**!
- I strongly encourage you to attend these unique lectures, as you will have an opportunity to learn latest skills and techniques for designing and implementing advanced intelligent systems.

What did we learn last time?

Requirements for KR

- So, how we *represent* knowledge in a form amenable to computer manipulation?
- **Desirable features** of KR scheme:
- *representational adequacy*;
- *inferential adequacy*;
- *inferential efficiency*;
- *well-defined syntax & semantics*;
- *naturalness*.

Logic

- Intelligent systems require that we have
  - Knowledge formally represented
  - New inferences/conclusions possible.
- Formal languages have been developed to support knowledge representation.
- One important one is the use of logic - very general purpose way to formally represent truths about the world, and draw sound conclusions from these.

Logic as a Knowledge Representation Language

- A Logic is a **formal language**, with **precisely defined syntax and semantics**, which **supports sound inference**. Independent of domain of application.
- Different logics exist, which allow you to represent different kinds of things, and which allow more or less efficient inference.
  - propositional logic, predicate logic, temporal logic, modal logic, description logic.
- All are defined by
  - syntax: what expressions are allowed in the language.
  - Semantics: what they mean, in terms of a mapping to real world
  - proof theory: how we can draw new conclusions from existing statements in the logic.
- Propositional logic is the simplest

Types of logic

Logics are characterized by what they commit to as "primitives"

Ontological commitment: what exists—facts? objects? time? beliefs?

Epistemological commitment: what states of knowledge?

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief $0 \ldots 1$
Fuzzy logic	degree of truth	degree of belief $0 \ldots 1$

Syntax and Semantics

- A *logic* usually has a well defined **syntax**, **semantics** and **proof theory**.
- The *syntax* of a logic defines the syntactically **acceptable objects of the logic**, or *well-formed formulae*.
- The *semantics* of a logic associate each formula with a *meaning*.
- The *proof theory* is concerned with manipulating formulae according to certain rules.

Propositional Logic: Syntax

- *propositions* and *connectives*.
- A **proposition** is a statement that is either true or false but not *both*
- *Logical connectives* are used to represent *and:* $\wedge$*, or:* $\vee$ *, if-then:* $\Rightarrow$*, not:* $\neg$*; T, F*

Syntax

- It is possible to determine whether any given statement is a proposition by prefixing it with
- *It is true that . . .*

and seeing whether the result makes grammatical sense.

- Propositions are often *abbreviated* using *propositional variables* eg p, q, r.
- Thus we **must associate the propositional variable with its meaning i.e.**

- Let p be *Tony Blair is Prime Minister*.

- Alternatively we could write something like

reactor_is_on

- so that the meaning of the propositional variable becomes obvious.

Syntax: Symbolic Representation

- p, q, r can stand for. . .

p – “Tony Blair is Prime Minister”

q – “2 + 3 = 5”

r – “ ‘Phone’ has five letters “

- 2 + 3 = 6
- ‘Work’ has five letters.
- the reactor is on

Not Propositions

- What’s the time?
- Oh bother!
- 2+3
- Look like propositions, but not quite...
- *All* elephants have 4 legs
- I like spinach

must associate the propositional variable with its meaning

Syntax: Well Formed Formulae

- Propositions may be combined with other propositions to form **compound propositions**. These in turn may be combined into further propositions.
- The set of sentences or *well-formed propositional formulae* (WFF) is defined as:
- Any propositional symbol is in WFF.
- The nullary connectives, **true** and **false** are in WFF.
- If A and B are in WFF then so is ¬A, A ^ B , etc

Examples

The following are well formed formulae

$$p \qquad (p) \qquad p \vee q \qquad (((p \Rightarrow q) \wedge \neg q) \Rightarrow \neg p)$$

whereas

$$(()) \qquad (p \wedge q) \vee \qquad (p \neg \Rightarrow q)$$

are not.

Propositional Logic: Semantics

- **What does it all mean?**
- Sentences in propositional logic tell you about what is true or false.
  - $P \wedge Q$ means that both P and Q are true.
  - $P \vee Q$ means that either P or Q is true (or both)
  - $P \Rightarrow Q$ means that if P is true, so is Q.
- This is all formally defined using *truth tables.*

X Y	*X v Y*
T T	*T*
T F	*T*
F T	*T*
F F	*F*

We now know exactly what is meant in terms of the truth of the elementary propositions when we get a sentence in the language (e.g., P => Q v R).

- The *conjunction* 'p AND q', written $p \wedge q$, of two propositions is true when both p and q are true, false otherwise.
- We can summarise the operation of $\wedge$ using a truth table. Rows in the table give all possible setting of the propositions to true (T) or false (F).

AND

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	Its Monday.
q	Its raining.
$p \wedge q$	Its Monday and its raining.
	Its Monday but its raining.
	Its Monday. Its raining.

Meaning

Caution:

p	I took a shower
q	I woke up
$p \wedge q$	'I took a shower and I woke up'
$q \wedge p$	'I woke up and I took a shower'.
Logically the same!	*WE* may see difference

The word *both* is often useful eg *I both took a shower and I woke up*.

Also called *disjunction*

The disjunction 'p OR q', written $p \vee q$, of two propositions is true when p or q (or both) are true, false otherwise.

Sometimes called *inclusive or*.

OR

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	Its Monday.
q	Its raining.
$p \vee q$	Its Monday or it is raining.

The word *either* is often useful eg *either its Monday or it is raining*.

It *also* includes the case of rain on a Monday!

The negation 'NOT p' of a proposition (or $\neg p$) is true when p is false and is false otherwise. $\neg p$ may be read that it is false that p.

p	$\neg p$
T	F
F	T

Negation is a *unary* connective. It only takes one argument. Conjunction and disjunction were both *binary* connectives.

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p Logic is easy.

$\neg p$ It is false that logic is easy.

It is not the case that logic is easy.

Logic is not easy.

If . . . then . . . $\Rightarrow$

Also known as **implication**

The implication 'p IMPLIES q', written $p \Rightarrow q$, of two propositions is true when either p is false or q is true, and false otherwise.

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	I study hard.
q	I get rich.
$p \Rightarrow q$	If I study hard then I get rich.
	Whenever I study hard, I get rich.
	That I study hard implies I get rich.
	I get rich, if I study hard.

MEANING

$p \Rightarrow q$ is true in the following situations

- I study hard and I get rich; or
- I don't study hard and I get rich; or
- I don't study hard and I don't get rich.

The last two situations, i.e. when p is false (I don't study hard) we can't say whether I will get rich or not. However if I've studied hard but failed to become rich then the proposition is clearly false.

Proof Theory and Inference

- So, let P mean “It is raining”, Q mean “I carry my umbrella”.
- If we know that P is true, and P => Q is true..
- We can conclude that Q is true.
- Note that certain expressions are equivalent
  – think about P => Q and $\neg$ P v Q.

$$\frac{P,\ P \Rightarrow Q}{Q}$$

Proof Theory

- How do we draw new conclusions from existing supplied facts?
- We can define **inference rules**, which are guaranteed to give true conclusions given true premises.
- For propositional logic useful one is **modus ponens**:
- If P is true and P=> Q is true, then conclude Q is true.

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Proof Theory

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The biconditional, written as $p \Leftrightarrow q$, of two propositions is true when both p and q are true or when both p and q are false, and false otherwise.

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Contrapositive

$p \Rightarrow q$	If I study hard then I get rich.
$q \Rightarrow p$	If I get rich then I study hard. (the **converse.**)
$\neg q \Rightarrow \neg p$	If I don't get rich then I don't study hard. (the **contrapositive.**)

Semantics of compound formulae

Construct a table:

- Construct a column for each proposition involved.
- Put given values for propositions into the table
- Construct a column for each connective, the most deeply nested first.
- Evaluate each column using values for propositions or previous columns.

More complex rules of inference

- Other rules of inference can be used, e.g.,:

$$\frac{A \vee B, \neg B \vee C}{A \vee C}$$

- This is essentially the *resolution* rule of inference, used in Prolog.
- Consider:
- What can we conclude?

sunny v raining
¬ raining v umbrella

Proof

- Suppose we want to try and prove that a certain proposition is true, given some sentences that are true.
- It turns out that the resolution rule is sufficient to do this.
- We put all the sentences into a standard or “normal” form (replacing A => B with $\neg$ A v B)
- There is then a standard procedure that lets you determine if the proposition in question is true (inferential adequacy).

Where are we…

- What is a logic?
- A formal language
  - Syntax – what expressions are legal (well-formed)
  - Semantics – what legal expressions mean
    - in logic the truth of each sentence with respect to each possible world.
- E.g the language of arithmetic
  - X+2 >= y is a sentence, x2+y is not a sentence
  - X+2 >= y is true in a world where x=7 and y =1
  - X+2 >= y is false in a world where x=0 and y =6

Entailment

- One thing follows from another

  $KB \models \alpha$

- KB entails sentence α *if and only if* α is true in worlds where KB is true.

  E.g. x+y=4 entails 4=x+y

- Entailment is a relationship between sentences that is based on semantics.

Example

- “WalMart defended itself in court today against claims that its female employees were kept out of jobs in management because they are women”

ENTAILS (‘IS SUBSUMED BY’)

“WalMart was sued for sexual discrimination”

Entailment

$$KB \models \alpha$$

Knowledge base KB entails sentence α
if and only if
α is true in all worlds where KB is true

E.g., the KB containing "the Giants won" and "the Reds won" entails "Either the Giants won or the Reds won"

Entailment is different than inference

Models

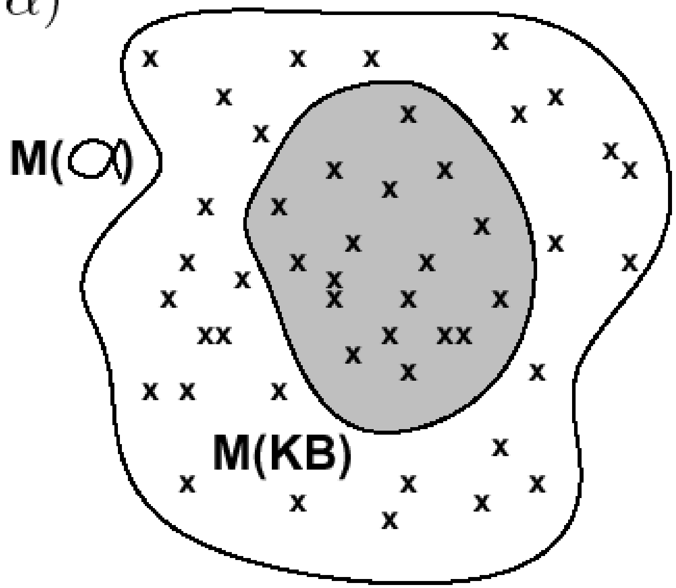
Logicians typically think in terms of <u>models</u>, which are formally structured worlds with respect to which truth can be evaluated

We say m is a <u>model</u> of a sentence α if α is true in m

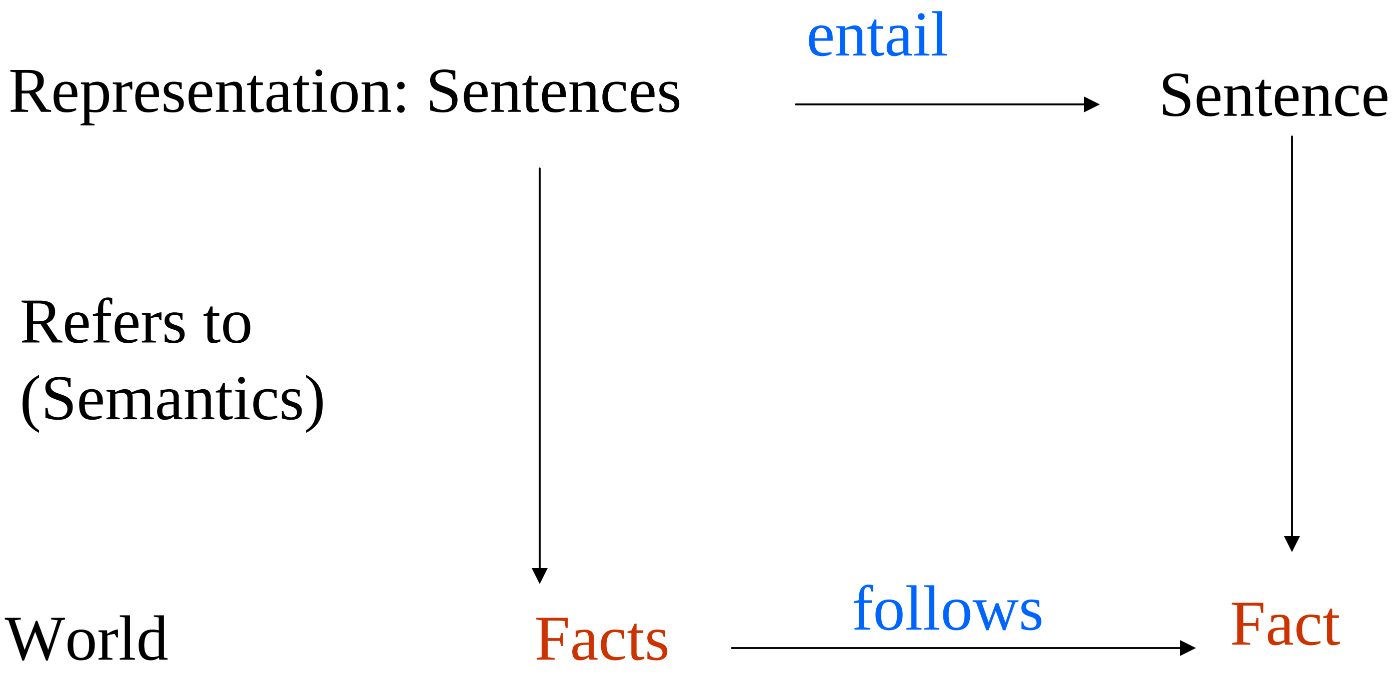
$M(\alpha)$ is the set of all models of α

Then $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$

E.g. $KB =$ Giants won and Reds won
$\alpha =$ Giants won



Logic as a representation of the World



Proof Theory

- Reasoning about statements of the logic without considering interpretations is known as *proof theory*.
- *Proof rules* (or inference rules) show us, given true statements how to generate further true statements.
- *Axioms* describe ‘universal truths’ of the logic.
  - Example $\vdash p \vee \neg p$ is an axiom of propositional logic.
- We use the symbol $\vdash$ denoting *is provable* or *is true*.
- We write $A_1, \ldots A_n \vdash B$ to show that B is provable from $A_1, \ldots A_n$ (given some set of inference rules).

Inference Rules

- Rules of inference can be used to deduce validity **without building a truth table**. Some simple inference rules:

And-elimination: From a conjunction you can infer any of the conjuncts.

$P1 \,\&\, P2 \,\&\, P3 \ldots \models Pi$

And-introduction: From a list of propositions you can infer their conjunction.

$P1, P2, P3 \models P1 \,\&\, P2 \,\&\, P3$

(This is a purely syntactic rule.)

Or-introduction: From a proposition you can infer its disjunction with anything else!

$P1 \models P1 \vee P2 \vee P3$

More Inference Rules

Double-negation elimination: From a doubly negated proposition you
can infer a positive proposition.

$\neg \neg P \models P.$

Unit resolution: If one disjunct is false, then you can infer that the other one is true.

$(P \vee Q) \& (\neg Q) \models P.$

Modus ponens: If the antecedent of an implication is true, then so is its consequent.

$((P \Rightarrow Q) \& P) \models Q$

Modus tollens: If the consequent of an implication is false, then so is its antecedent.

$((P \Rightarrow Q) \& \neg Q) \models \neg P$

Yet More Rules

Chain rule: Implication is transitive

$((P \Rightarrow Q) \,\&\, (Q \Rightarrow r)) \models (P \Rightarrow r)$

or

$(\neg P \vee Q) \& (\neg Q \vee r) \models (\neg P \vee r)$

Resolution: Alternate form of the chain rule.

$(P \vee Q) \& (\neg Q \vee R) \models (P \vee R)$

The intuition is that Q cannot be both true and false, so one or the other disjuncts must be true.

Resolution is the inference rule used in most automatic theorem proving systems